Quiz 4A - MTH 1410

10:37
10:41
4

Name: Key

1) (4 points) Use logarithmic differentiation to find $\frac{dy}{dx}$ if $y = x^{\sqrt[4]{x}}$. Simplify your answer.

$$\ln y = \ln x^{(x^{1/4})} \quad \checkmark$$

$$\frac{d}{dx}\left(\ln y = x^{1/4} \cdot (\ln x)\right) \quad \checkmark\checkmark$$

$$\frac{1}{y} \cdot y' = \frac{1}{4} x^{-3/4} \cdot \ln x + \frac{1}{x} \cdot x^{\frac{1}{4}} \quad \checkmark\checkmark\checkmark$$

$$y' = y\left(\frac{\ln x}{4x^{3/4}} + x^{-3/4}\right) \quad \checkmark$$

$$y' = x^{\sqrt[4]{x}} \cdot x^{-3/4}\left(\frac{\ln x}{4} + 1\right) \quad \checkmark$$

$$y' = x^{\sqrt[4]{x} - 3/4}\left(\frac{\ln x}{4} + 1\right)$$

2) (3 points) Find $f'(x)$ if $f(x) = \sin(\arccos x)$. Simplify your answer.

$f'(x) = \cos(\arccos x) \cdot \frac{-1}{\sqrt{1-x^2}}$ ✓ ✓

$\Rightarrow f'(x) = x \cdot \frac{-1}{\sqrt{1-x^2}}$ ✓

$\Rightarrow f'(x) = \boxed{\frac{-x}{\sqrt{1-x^2}}}$

3) (3 points) A stone is thrown vertically upward and its height after t seconds in $96t - 16t^2$. Use calculus to find the maximum height reached by the stone.

$h(t) = 96t - 16t^2$ ✓

$h'(t) = 96 - 32t$ ✓

Max height when $h'(t) = 0$ ✓

$0 = 96 - 32t \Rightarrow 96 = 32t$

$3 = t$ ✓

$h(3) = 96 \cdot 3 - 16 \cdot 3^2$

$= 288 - 144 = \boxed{144}$ ✓

Quiz 4B - MTH 1410

Name: Key

1) (3 points) Find $f'(x)$ if $f(x) = \sin(\arccos x)$. Simplify your answer.

$$f'(x) = \cos(\cos^{-1}x) \cdot \frac{-1}{\sqrt{1-x^2}}$$

$$= x \cdot \left(\frac{-1}{\sqrt{1-x^2}}\right)$$

$$f'(x) = \frac{-x}{\sqrt{1-x^2}}$$

2) (3 points) A stone is thrown vertically upward and its height after t seconds in $128t - 16t^2$. Use calculus to find the maximum height reached by the stone.

$f(t) = 128t - 16t^2$ ✓

$f'(t) = 128 - 32t$ ✓

at max height when $f'(t) = 0$ ✓

$\Rightarrow 0 = 128 - 32t$

$\Rightarrow 32t = 128$

$t = 4$ ✓

Max height $= f(4) = 128 \cdot 4 - 16(4^2)$

$= 512 - 256$

✓✓ $= \boxed{256}$

3) (4 points) Use logarithmic differentiation to find $\frac{dy}{dx}$ if $y = x^{\sqrt[3]{x}}$.

$\ln y = \ln x^{\sqrt[3]{x}}$ ✓

$\Rightarrow \ln y = x^{1/3} \cdot (\ln x)$ ✓

$\Rightarrow \frac{d}{dx}\left(\ln y = x^{1/3} \cdot (\ln x)\right)$ ✓

$\Rightarrow \frac{1}{y} \cdot y' = \frac{1}{3} x^{-2/3} \cdot \ln x + \frac{1}{x} \cdot x^{1/3}$ ✓✓✓

$y' = y \cdot \left(\frac{1}{3} x^{-2/3} \ln x + x^{-2/3}\right)$ ✓

$\boxed{y' = x^{\sqrt[3]{x}} \cdot x^{-2/3}\left(\frac{1}{3}\ln x + 1\right)}$ ✓

$\boxed{y' = x^{\sqrt[3]{x} - 2/3}\left(\frac{1}{3}\ln x + 1\right)}$